

A DIDACTICAL CONCEPT FOR THE COMPUTER-AIDED DEMONSTRATION OF DIFFERENT WAYS OF PROJECTION USED IN DESCRIPTIVE GEOMETRY

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ABSTRACT

It is one task of Descriptive Geometry to assess the different ways of projection in view of their suitability to show spatial objects with regard to the extent of the realistic effect of the drawing. To aid the learning of this ability while teaching we developed a concept for the computer-aided demonstration of different ways of projection used in Descriptive Geometry. As the result we present a program which uses a simple house (or tower) with sloped roofs to demonstrate the influence on the realistic effect of a drawing caused by different ways of projection, different directions of the projective beams and different positions of the image plane. Key words: Descriptive Geometry, projection, computer-aided demonstration.

1. INTRODUCTION

Drawings with realistic results can be achieved by PARALLEL PROJECTION or by CENTRAL PROJECTION. The more the projective beams and the visual rays (of the observer) correspond, the better is the realistic effect of a drawing. More precisely: Looking onto a drawing, the bundle of the visual rays and the bundle of the projective beams should differ as little as possible. The degree of this correspondence varies with different ways of projection and is determined by the position of the observer. As the construction of different drawings causes different effort it is useful to estimate beforehand, which way of projection will generate the best result for the special object. For the realistic effect the correspondence of the projective beams and the visual rays is of particular interest. In practice the difficulty to find the correct position of the eye for different ways of projection differs. Teaching Descriptive Geometry it has to be shown if and to which extent the correspondence of the projective beams and the visual rays can be achieved with different ways of projection. One aspect for the valuation is the extent of the apparent variance of the proportions by the projection.

We designed the program in a way, that the influence of different ways of projection can be directly controlled. Moreover for each way of projection the program easily demonstrates from which positions the object can be shown.

At the RWTH Aachen this program is used teaching "Descriptive Geometry for architects" and "Descriptive Geometry for civil-engineers". In bigger classes the usage of the program with a laptop and an overhead display is advisable.

2. MANUAL OF THE PROGRAMM

The following manual of the program DG-RWTH gives an idea of the variety of its usage in teaching Descriptive Geometry. It will be shown how the program allows to make a choice of different ways of projection, different directions of the projective beams and different positions of the image plane. The main advantage of the program, the possibility of interactive working and the direct change of the projective beams or the position of the image plane, can only be experienced while actually using the program.

2.1 Choice of the Way of Projection

In the context of Descriptive Geometry parallel projection and central projection are often used. (The object correlates with a spatial, orthogonal, positive-oriented coordinate system in a way, that the z -axis following the gravitation is vertical, the x,y -plane (ground-plan-plane) is therefore horizontal (cf. Fig. 2). The y,z -plane will be called elevation-plane.)

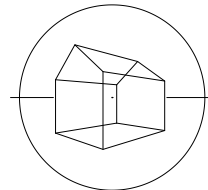


Fig. 1

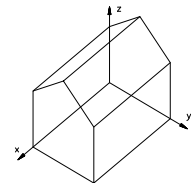


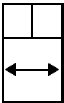
Fig. 2

C In **CENTRAL PROJECTION** (cf. Fig. 1) all projective beams intersect in one point O , the center of projection (eye). In the program central projection is chosen with the key **C**.

P In **PARALLEL PROJECTION** (cf. Fig. 2) all projective beams are parallel. Parallel projection is a special case of central projection where the center of projection is at infinity. In the program parallel projection is chosen with the key **P**. Moreover starting in central projection, the distance between the eye and the object can be advanced by moving the mouse (see below) in a way, that the program changes to parallel projection.

2.2 Choice of the Direction of View

The direction of the projective beams is crucial for the picture of the object and decides, from which side the observer will see the object respectively. The program allows for **PARALLEL PROJECTION** (**P**) to change the direction of the view (the projective beams) using the mouse. (In this text a movement with the mouse to the left or the right will be called horizontal, the perpendicular movement will be called vertical.)

 By horizontal mouse movement (without pressing a key) the projective beams are turned around the vertical axis of the house, and the observer walks around the house respectively.

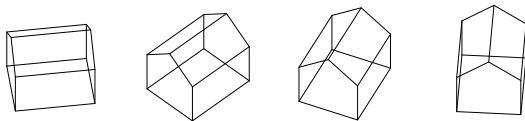
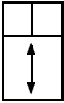


Fig. 3 Fig. 4 Fig. 5 Fig. 6

 By vertical mouse movement (without pressing a key) the angle of the projective beams against the horizontal plane is changed, i.e. the observer is looking more from above and more from below onto the house respectively.

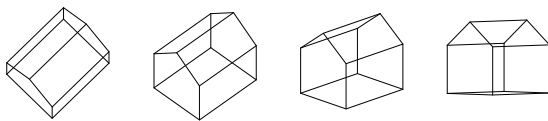
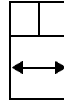


Fig. 7 Fig. 8 Fig. 9 Fig. 10

In **CENTRAL PROJECTION** (**C**) the direction of the view is determined by the eye's position (center of projection); in the program the position can be changed by using the mouse. (In order to allow the direct comparison of the different effects of parallel projection and central projection the program treats the direction of the main visual ray like the direction of the projective beams. The main visual ray is the ray through the eye, which is perpendicular to the image plane.) For additional orientation the circle of sight is shown; the circle of sight is the intersection of the image plane with

a cone of visual rays, the axis of the cone being the main visual ray and the angle between the axis and the visual rays being 30° .

 By horizontal mouse movement (without pressing a key) the main visual ray is turned around the vertical axis of the house, and the observer walks around the house respectively.

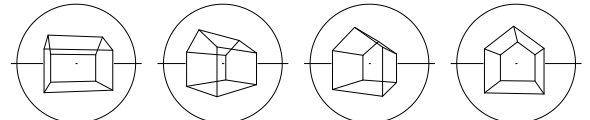
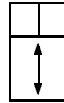


Fig. 11 Fig. 12 Fig. 13 Fig. 14

 By vertical mouse movement (without pressing a key) the angle of the main visual ray against the horizontal plane is changed, i.e. the observer is looking more from above or more from below onto the house respectively.

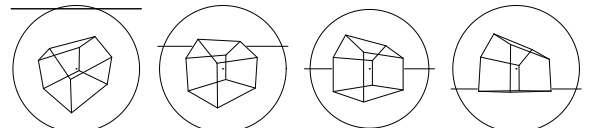
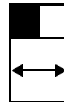


Fig. 15 Fig. 16 Fig. 17 Fig. 18

In addition to the change of the direction of the main visual ray in central projection further parameters can be modified.

 By horizontal mouse movement with the left key pressed the eye is moved accordingly to the side.

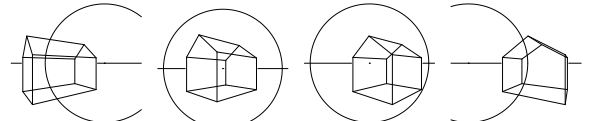
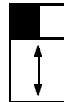


Fig. 19 Fig. 20 Fig. 21 Fig. 22

 By vertical mouse movement with the left key pressed the height of the eye is changed accordingly.

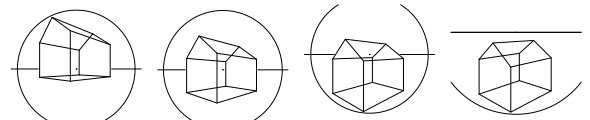
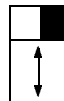


Fig. 23 Fig. 24 Fig. 25 Fig. 26

 By vertical mouse movement with the right key pressed the distance between the eye and the house is changed, i.e. the observer is either moving closer to the house or away from it. If the distance gets *very large*, the program changes to parallel projection. Vice versa parallel projection can be changed to central projection by pressing the right mouse key.

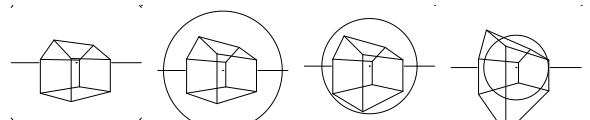
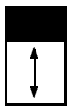


Fig. 27 Fig. 28 Fig. 29 Fig. 30



By vertical mouse movement with the left and the right key pressed the distance between the image plane and the eye is changed, i.e. the size of the image is reduced or enlarged.

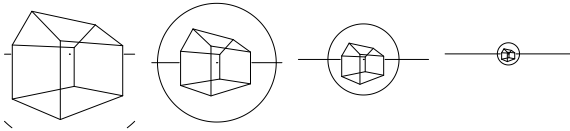


Fig. 31

Fig. 32

Fig. 33

Fig. 34

The following additional functions for the choice of the view direction are implemented in the program:



Those movements which are possible by mouse movements can be passed through automatically. Pressing the key again this function is closed. The velocity of the “turning“ will be automatically adjusted to the cpu-power.



Pressing this key the angle between the projective beams and the horizontal plane is set to 45° (This function is designed for the demonstration of the **OBLIQUE VIEW ONTO A HORIZONTAL IMAGE PLANE**, if only this view angle is adequate); by moving the mouse only the horizontal angle will be modified. Pressing the key **M** (see below) the angle between the projective beams and the horizontal plane can be changed again by moving the mouse.



The projective beams are set to the direction which is needed for the **ISOMETRY**.



Pressing this key the direction of the view is reset to the position which was handled after the last pressing of a key.



Pressing this key twice all parameters are reset to the positions which were handled when the program started.



After having pressed this key only horizontal mouse movements are regarded; vertical components will be ignored.



After having pressed this key only vertical mouse movements are regarded; horizontal components will be ignored.



After having pressed this key all mouse movements are regarded again.



After having pressed both keys simultaneously special projective beams will be caught if the mouse is moved; e.g. angles from -3° till $+3^\circ$ will be set automatically to 0° . Pressing both keys again this function is closed.



Pressing both keys simultaneously a **PERSPECTIVE ONTO AN INCLINED IMAGE PLANE** (main visual ray pointing upward) will be shown.



After having pressed both keys simultaneously a tower is shown instead of the house; pressing the key again this function is closed.

2.3 Choice of the Position of the Image Plane

Depending on the position of the image plane in relation to the object, and to the coordinate system respectively, which the object is related to, there are different ways of projection used in Descriptive Geometry. The following possibilities are distinguished:



In default or pressing the key **O** the image plane will be positioned orthogonal (perpendicular) to the beams of projection and the main visual ray respectively (Depending on the chosen view direction an unfavourable image might emerge; pressing the keys **Ctrl O** simultaneously the view direction is appropriately changed.):

In **PARALLEL PROJECTION** **P** the following cases occur:

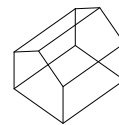


Fig. 35

If the projective beams are inclined, the result is an **ORTHOGRAPHIC VIEW ONTO AN INCLINED IMAGE PLANE**.

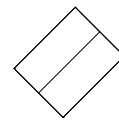


Fig. 36

If the projective beams are vertical, the result is a **GROUND-PLAN** (orthographic view onto a horizontal image plane).



Fig. 37

If the projective beams are horizontal and perpendicular to the y,z -plane, the result is an **ELEVATION** (orthographic view onto a vertical image plane).

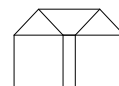


Fig. 38

If the projective beams are in general horizontal, the result is a **SIDE VIEW** (orthographic view onto a vertical image plane).

In **CENTRAL PROJECTION** **C** the following cases occur:

(In central projection the center of projection has to stay away from the image plane, if a realistic image is to be achieved, because otherwise the image of all points, which are not part of the image plane, is only one point. Moreover the program will compute no image, if a part of the house lies beyond the plane of vanishing. The center of projection is part of the plane of vanishing, which is parallel to the image plane.)

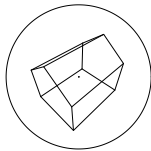


Fig. 39

If the main visual ray is inclined, the result is a **PERSPECTIVE ONTO AN INCLINED IMAGE PLANE**.

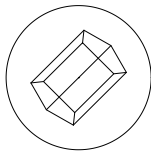


Fig. 40

If the main visual ray is vertical, the result is a **PERSPECTIVE ONTO A HORIZONTAL IMAGE PLANE**.

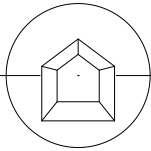


Fig. 41

If the main visual ray is horizontal and perpendicular to the y,z -plane, the result is a **FRONTAL PERSPECTIVE** (perspective onto a special vertical image plane).

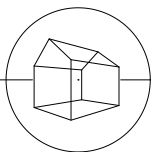


Fig. 42

If the main visual ray is generally horizontal, the result is a **PERSPECTIVE** (onto a vertical image plane).

G

Pressing this key the image plane will be positioned horizontal, therefore parallel to the image plane of the ground-plan (Depending on the chosen view direction an unfavourable image might emerge; pressing the keys **[Ctrl]** **[G]** simultaneously the view direction is appropriately changed.):

In **PARALLEL PROJECTION** **[P]** the following cases occur:

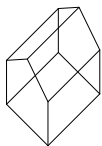


Fig. 43

If the projective beams are inclined, the result is a **BIRDS-EYE-VIEW** (oblique view onto an inclined image plane).

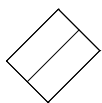


Fig. 44

If the projective beams are vertical, the result is a **GROUND-PLAN** (orthographic view onto a horizontal image plane).

Fig. 45

If the projective beams are horizontal, there is no result.

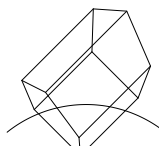


Fig. 46

In **CENTRAL PROJECTION** **[C]** the main visual ray is always perpendicular to the image plane, in this case it is therefore vertical; the result is a **PERSPECTIVE ONTO A HORIZONTAL IMAGE PLANE**.

E

Pressing this key the image plane will be positioned parallel to the y,z -plane (elevation-plane) (Depending on the chosen view direction an unfavourable image might emerge; pressing the keys **[Ctrl]** **[E]** simultaneously the view direction is appropriately changed.):

In **PARALLEL PROJECTION** **[P]** the following cases occur:

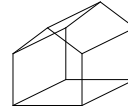


Fig. 47

If the projective beams are inclined, the result is a **CAVALIER-PERSPECTIVE** (oblique view onto a vertical image plane).



Fig. 48

If the projective beams are horizontal, the result is an **ELEVATION** (orthographic view onto a vertical image plane).

Fig. 49

If the projective beams are vertical, there is no result.

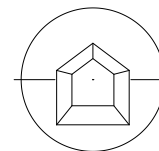


Fig. 50

In **CENTRAL PROJECTION** **[C]** the main visual ray is always perpendicular to the image plane, in this case it is therefore horizontal; the result is a **FRONTAL PERSPECTIVE** (perspective onto a vertical image plane).

V

Pressing this key the image plane will be positioned generally vertical (Depending on the chosen view direction an unfavourable image might emerge; pressing the keys **[Ctrl]** **[V]** simultaneously the view direction is appropriately changed.):

In **PARALLEL PROJECTION** **[P]** the following cases occur:

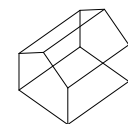


Fig. 51

If the projective beams are inclined, the result is a **GENERAL OBLIQUE VIEW ONTO A VERTICAL IMAGE PLANE**.

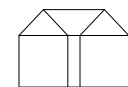


Fig. 52

If the projective beams are perpendicular to the image plane, the result is a **SIDE VIEW** (orthographic view onto a vertical image plane).

Fig. 53

If the projective beams are vertical, there is no result.

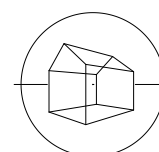


Fig. 54

In **CENTRAL PROJECTION** **[C]** the main visual ray is always perpendicular to the image plane, in this case it is therefore horizontal; the result is a **PERSPECTIVE** (onto a vertical image plane).

2.4 Choice of the Perspective Parameters

A special part of the program is designed to demonstrate the necessary steps to choose the parameters of a perspective onto a vertical image plane for manual construction.

W This part of the program is activated with the key **W** and is left by pressing this key again. (Within this part of the program, it is not necessary to press any keys while moving the mouse.)

0 Pointing the Main Visual Ray onto the House: After having pressed the key **0** the main visual ray is relocated accordingly to the side by horizontal mouse movement.

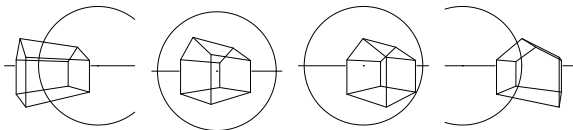


Fig. 55 Fig. 56 Fig. 57 Fig. 58

1 Choice of the Main Visual Ray in the Ground-plan:

After having pressed the key **1** the main visual ray is turned around the vertical axis of the house (i.e. the observer walks around the house) by horizontal mouse movement.

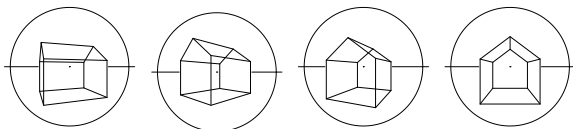


Fig. 59 Fig. 60 Fig. 61 Fig. 62

2 Choice of the Main Visual Rays Height:

After having pressed the key **2** the height of the main visual ray is changed by vertical mouse movement.

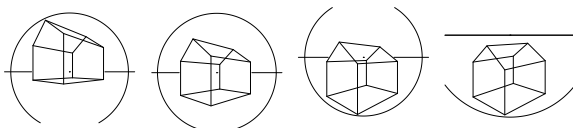


Fig. 63 Fig. 64 Fig. 65 Fig. 66

3 Choice of the Eye on the Main Visual Ray (distance between the eye and the house):

After having pressed the key **3** the distance between the eye and the house is changed (i.e. the observer is moving closer to the house or is moving away from it) by vertical mouse movement.

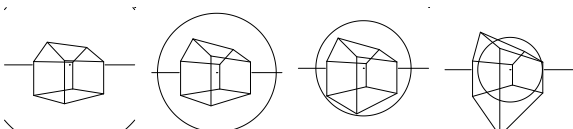


Fig. 67 Fig. 68 Fig. 69 Fig. 70

4 Choice of the Image Planes Position (largeness of the image):

After having pressed the key **4** the distance between the image plane and the eye is changed (i.e. the size of the image is reduced or enlarged) by vertical mouse movement.

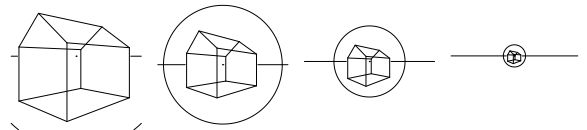


Fig. 71 Fig. 72 Fig. 73 Fig. 74

2.5 Additional Functions

? Pressing this key a help menu appears on the screen.

Ctrl L After having pressed both keys simultaneously the circle of sight, which happens to appear as an ellipse on some screens, can be corrected by pressing the keys **-** i.e. **+**. (Modification of the aspect ratio)

Ctrl P After having pressed both keys simultaneously the image is "printed", i.e. a file with HPGL-commands (Hewlett Packard Graphics Language) is generated.

Ctrl A Pressing both keys simultaneously the parameters for the angle of the projective beams or the position of the eye can be specified by keyboard program input.

Q Pressing this key the program is closed.

This program can be found in the internet:

<http://www.igpm.rwth-aachen.de/dg/DG-RWTH.html>

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